OPTIMAL CONTROL USING A STATE OBSERVER CONSIDERING DISTURBANCES OF A GOLF SWING ROBOT

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ABSTRACT

This paper studies an optimal control of a golf swing robot that is used to evaluate the performance of golf clubs. Mathematical model of the golf club is derived by Hamilton's principle in consideration of the bending and torsional stiffness, and the eccentricity of gravity center of the club head on the shaft axis. The state variables that cannot be measured are estimated by state observer considering disturbance. Control torque is applied after the impact. LQR is applied to stop the robot during the follow-through. Numerical simulation is compared with experimental results by using a swing robot.

1. INTRODUCTION

Since each joint of most golf swing robots is braked down to stop forcibly after the downswing, golf clubs and swing robots are vibrated by the shock of braking and the accuracy of the evaluation results falls off. Although many researchers have studied torque plans for the golf robot to realize actual human swings [1,2], reduction methods of the shock and vibration in the finish of the swing have not been studied enough.

In this study, an optimal control is applied to the swing robot to suppress the shock and vibration due to braking after the downswing. That robot can be thought as a kind of fast motion manipulators with a flexible link. Then we assume in this paper that the robot has two joints, a rigid link and a flexible link. The swing robot pulls up the golf club (back-swing) and then swings it in a swing plane. The golf club is modeled as a flexible beam in consideration of the bending and torsional stiffness, and the eccentricity of gravity center of the club head on the shaft axis. Because of the boundary conditions of the club head, the bending in the swing plane and the torsional vibration of the club shaft are coupled. The natural frequencies and the vibration modes are obtained by numerical and experimental modal analysis. LQR considering the vibration of the club shaft is applied during the follow-through to stop the robot and reduce the shock. Since it is difficult to measure all of the state variables, a state observer is employed to the state feedback control system. The robot manipulators are affected strongly from Coriolis and centrifugal forces in fast motion. In this case, an ordinary state observer for the linear system cannot estimate the states of the system correctly because of dynamics not modeled or uncertainty due to those nonlinear forces. To overcome this problem we apply a state observer considering disturbances to improve the performance of the state feedback control.

2. ASSUMPTION AND MODELING

The system of orthogonal axes \((x, y, z)\) is taken as figure 1, and the robot swings a golf club in a swing plane \(o-xy\). The angle between the direction of gravity and \(x\)-axis is \(a\) [rad]. The angle \(a\) depends on physique of golfers and length of golf clubs. The golf club is modeled as a flexible beam in consideration of the bending and torsional stiffness, and the eccentricity of gravity center of the club head on the shaft axis. The club-shaft is assumed to be homogeneous and initially straight along the \(\xi_2\)-axis shown in figure 3. The flexible club-shaft is cantilevered at the rigid grip link and properties of the shaft are as follows: a uniform cross-sectional area \(A_3\), bending stiffness \(E_3I_3\),

![Fig. 1 Swing Plane of the Golf Swing](image-url)
torsional stiffness $G_3 I_3$, mass per unit volume $\rho_3$ and the eccentricity of gravity center of the club head $L_4$. We assume that the robot has two joints, two rigid links that are modeled on the arm of the golfer and the grip of the club. The club-shaft is modeled as a flexible link. The robot has a hook mechanism to lock the wrist joint [1,2]. The swing robot for experiment is shown in figure 2. Figure 3 shows the coordinate system in the xy-plane. Angles $\theta_1$ and $\theta_2$ are the absolute rotating angles of the first joint and the second joint respectively.

$L$ is Lagrangian of the system defined by

$$L = T - U.$$  

where $T_1$, $T_2$ and $T_4$ are kinetic energies of the first link, second link and tip mass, respectively and $t_3$ is kinetic energy per unit length of the flexible link. Potential energy of the system is given by

$$U = U_1 + U_2 + \int_0^t u_3 d\xi + U_4;$$

where $U_1$, $U_2$ and $U_4$ are potential energies of the first link, second link and tip mass, respectively and $u_3$ is potential energy per unit length of the flexible link. Hamilton’s principle is applied to derive the equations of motion and the boundary conditions.

Ignoring the non-linear terms, the boundary conditions of the club head are obtained as

$$m_4 \ddot{v}_\eta(L_4) - \dot{\phi}(L_4)m_4 L_4 - E_2 l_3 v''_\eta(L_3) = 0,$$

$$J_{4c} \ddot{v}'_\eta(L_3) + E_2 l_3 v''_\eta(L_3) = 0,$$

and

$$-m_4 L_4 \ddot{v}_\eta(L_4) + (J_{4c} + m_4 L_4^2) \dot{\phi}(L_4) + G_3 I_3 \phi(L_3) = 0.$$

From these equations, we can know that the bending vibration in the swing plane and the torsional one are coupled. Figure 4 shows the comparison of the natural frequencies of the bending-torsion coupling model and the independent model. Using a same time function $q(t)$, we assume the bending displacement $v_\eta$ and the torsional angle $\phi$ as

$$\begin{cases} v_\eta(\xi,t) = q(t) \Gamma_\eta(\xi) \\ \phi(\xi,t) = q(t) \Gamma_\phi(\xi) \end{cases}$$

where $\Gamma_\eta$ and $\Gamma_\phi$ are the mode shapes of bending and torsional angle, respectively. The natural frequencies and the vibration modes are obtained by numerical and experimental modal analysis. The bending displacement $v_\eta$ and the torsional angle $\phi$ can be expressed as
An approximate system of finite dimensional equations is obtained using a finite number of modes in equation (8). Only the first two modes are considered in this paper. This model describes the dynamics adequately within the frequency bandwidth of interest. Applying orthogonality relationships of modal functions for the equations of motion, the non-linear equations of motion of the finite dimensional model are given by

\[ J(x) \ddot{x} + D \dot{x} + Kx + h(x, \dot{x}) + g(x) - pu = 0 \]  

where

\[ x = [\theta_1, \psi, q_1, q_2]^T, \quad u = [\tau_1, \tau_2]^T, \quad p = [I_{2x2} \quad 0_{2x2}]^T \]

and \( J, D \) and \( K \) are inertia, damping and rigidity matrices, and \( h, g \) and \( u \) are nonlinear force, gravity and input vectors respectively.

**Controller design**

Ignoring the gravity in equation (9), linearizing around \( x=0 \) and substitutively considering the disturbance vector \( d \), we can obtain the linear equations of motion as

\[ J(t) \ddot{x} + D(t) \dot{x} + Kx + h(x, \dot{x}) + g(x) - pu = d \]  

Equation (10) can be rewritten as the state variable equation

\[ \dot{X} = AX + Bu + Ed \]  

where

\[ X = [x^T \quad \dot{x}^T]^T, \quad A = \begin{bmatrix} 0 & I \\ -J^{-1}K & -J^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ J^{-1}P \end{bmatrix} \text{ and } E = \begin{bmatrix} 0 \\ J^{-1} \end{bmatrix}. \]

The output equation is

\[ Y = [\gamma \quad \theta_1 \quad \psi]^T = CX \]  

where

\[ C = \begin{bmatrix} 0_{2x2} & r_{v_{\gamma}}(\xi_5) & r_{v_{\theta_1}}(\xi_5) & 0_{2x2} \\ J_{2x2} & 0_{2x2} \end{bmatrix} \]

Output \( \gamma \) is the shear strain due to the torsion of the flexible link and \( \xi_5 \) is the position of strain gauges. Discrete-time system of the equation (11) is given as

\[ X_{k+1} = A_dX_k + B_du_k + E_dd_k. \]  

Control input is described as

\[ u_k = -FX_k \]  

where \( F \) is LQR feedback gain matrix that minimizes the cost function

\[ J = \frac{1}{2} \sum_{k=1}^{\infty} [X_k^TQX_k + u_k^TRu_k]. \]

Because not all state variables can be measured directly, it is necessary for the perfect state feedback to estimate some state variables. In this paper, the state variables are estimated by a state observer mentioned in follow subsection.

**Observer design**

The robot manipulators are affected strongly from Coriolis and centrifugal forces in fast motion. An ordinary state observer for the linear system cannot estimate the states of the system correctly because of the uncertainty due to those nonlinear forces. The observer estimating a harmonic disturbance has been used in reference [3] and estimated disturbance has been fed forward to the system to cancel actual disturbance.

In this paper, for the reason that the dynamics of the disturbance is hard to be modeled, the dynamics of the disturbance vector \( d_k \) is assumed as

\[ \begin{cases} \eta_{k+1} = \Gamma \eta_k \\ d_k = H\eta_k \end{cases} \]

where \( \Gamma = H = I_{4x4} \).

Equation (13) and (15) can be combined into the state variable equation as

\[ \begin{cases} X'_{k+1} = \tilde{A}X'_k + \tilde{B}u_k \\ Y'_k = \tilde{C}X'_k \end{cases} \]

where

\[ \tilde{X}'_k = \begin{bmatrix} X_k \\ \eta_k \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A_d & E_d \Gamma \\ 0 & \Gamma \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \]

and
\( \tilde{C} = [C \ 0] \).

Then we simply construct a full-order observer,
\[ \dot{\hat{X}}_i = \bar{A} \hat{X}_i + \bar{B} u_i + F_0 (Y_i - \tilde{C} \hat{X}_i), \]  
(17)
of the nominal system described as equation (16) to estimate the state vector
\[ \hat{X}_i = \begin{bmatrix} \hat{Y}_i \\ \hat{\eta}_i \end{bmatrix}. \]
Then the estimated disturbance vector \( \hat{w}_k \) can be obtained as
\[ \hat{w}_k = \hat{d}_k + H \hat{\eta}_k, \]
in addition that this observer estimates the states correctly. By the way, equation (17) can be rewritten as
\[ \begin{bmatrix} \dot{\hat{X}}_{k+1} \\ \hat{w}_{k+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \hat{X}_k \\ \hat{w}_k \end{bmatrix} + \begin{bmatrix} E & F_0 \end{bmatrix} \begin{bmatrix} u_k \\ e_k \end{bmatrix} \]
(18)
and \( \hat{w}_k \) can be solved as
\[ \hat{w}_k = F_{o2} \sum_{i=1}^{k} e_{i+1} + w_0. \]  
(19)
The block diagram of this observer is shown in Figure 5. Because the feedback in the observer includes integrator as equation (19), it may be thought that this observer is type 1 digital servo system tracking the actual one. Moreover, this observer has a property like a low-pass filter for the sensors. This property has some advantages, because measurement noise is usually large in higher frequency region. Therefore, this observer is hard to be affected from the sensor noise, and can estimate the states correctly in lower frequency region, and its bandwidth is decided by \( F_{o2} \). Now equation (14) is rewritten as
\[ u_k = -F \dot{\hat{X}}_k \]  
(20)
then estimated states \( \dot{\hat{X}}_k \) is fed back to the system.

3. SIMULATION AND EXPERIMENT
The Swing robot has two links and its joints are driven by two AC servomotors. The shoulder joint consists of a 100[W] AC servomotor that is reduced by 1:10 ball reducer. The wrist joint motor consists of a 50[W] AC servomotor and it drives the wrist joint directly. The rotating angle of each joint is measured with the incremental encoder on each motor axis whose resolution is 2000 counts per revolution. The wrist joint has a mechanical hook that constrains the relative angle of the wrist joint in \( \psi \geq -90^\circ \) and the hook can achieve wrist cock action in the down swing [1,2]. A flexible solid beam made of ABS resin is cantilevered on the second
link. Shear strain due to the torsional deformation is measured with four strain gauges where bending and elongation are canceled by a bridge circuit. The controller is implemented on a DSP mounted on a PCI bus of a PC/AT computer and its sampling rate is 1[kHz].

Swing motion procedure is described as follows. Firstly, the robot tracks to the S type position profile reference signal shown in figure 6 in the back swing action by type 1 digital servo-system. Secondly, the triangular feed forward torque shown as figure 7 is added to only the shoulder joint to make the robot swing down [1,2]. Lastly, the robot is stopped with LQR as equation (20) in follow through. Suitable weight matrices $Q$, $R$ and feedback gain $F_o$ are decided by using the simulation results in this paper. Here quantization errors have been taken into account in the simulations.

Simulation results of displacement of the club-head are shown in figure 8 where the swing robot is stopped with the mechanical brake or LQR. When the robot is stopped with mechanical brake, the amplitude of the club shaft vibration is so large. However, the vibration is suppressed when the robot is stopped with LQR considering the flexibility of the club shaft. Figure 9 shows the comparison of the numerical simulation of the actual and the estimated angular velocities of the wrist.

![Fig. 8 Comparison of the Displacements of Club-head using the Mechanical Brake or LQR](image)

![Fig. 9 Angular Velocity of the Wrist Joint that is estimated by Minimal Order Observer](image)

![Fig. 10 Simulation Results of the Actual and the Estimated Angular Velocities of the Second Joint](image)

![Fig. 11 Simulation Results of the Estimated State Variable $q_2$ and the Actual Shear Strain](image)
joint in which the robot is stopped with using minimal order observer. The constrained force of the hook mechanism, the torque due to the gravity and the moment of inertia of the arm, that change non-linearly due to the angle of the joint, cause the modeling errors between the nonlinear real model and the observer for the linear one. Because of those errors, the observer cannot estimate the correct states and the performance of states feedback control becomes worse. The observer considering disturbances, however, can estimate the states more correctly. Figure 10 and 11 show simulation results obtained by using this observer. Experimental results are shown in figure 12 and 13. In these figures, the upper figures are through the swing motion and the lower figures show the results focused in the region around the down swing. The observer can estimate the states correctly in spite of above-mentioned modeling errors in these figures, and these experimental results agree with simulation results.

We can see that the observer considering disturbances estimates the correct states. The feedback gains for the states estimated by them can be decided easier than the gains for the states estimated by minimal order observer, then the control performance can be improved to stop the robot and to suppress the vibration of the flexible link. We expect that the control performance will be improved better by feeding forward estimated disturbances.

4. CONCLUSION

LQR considering the vibration of the club shaft is applied during the follow-through to stop the robot. The state observer considering disturbances estimates the states correctly in spite of non-linear torques due to the gravity, the mechanical hook and the moment of inertia. The feedback gains for the states estimated by them can be decided easier than the gains for the states estimated by minimal order observer, then the control performance to stop the robot and to suppress the vibration of the flexible link is improved.

REFERENCES

