Disturbance cancellation control of elastic supported cylinder that adapts to the frequency variation in vortex-induced vibration

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This paper studies the disturbance cancellation control to suppress a vortex-induced vibration of a circular cylinder with frequency variation in a cross flow. Fluctuating lift force in a vortex-induced cylinder is assumed as a disturbance and an observer to estimate the disturbance is applied to control the vibration of the cylinder. When the frequency of the observer differs from that of the disturbance, control performance may become worse. To control the cylinder in vortex-induced vibration even if the frequency and the lift force varies with the flow velocity, we attempt to remodel the observer with the frequency measured by FFT analysis at regular time intervals.

1. Introduction

Vortex-induced vibration of a circular cylinder is one of the fundamental problems in flow-induced vibration. Much effort has been made to investigate the mechanism of the vortex-induced vibration and many aspect of it has been revealed [1-4]. Most dangerous problem occurs when the vortex shedding frequency coincides with the natural frequency of the system. In this part, many researchers have tried to restrain the vortex shedding with passive and active mechanisms. In former approach, they applied tripping wires and splitters or changed the shape of the cross section of the cylinder to prevent the creation of vortices. In latter approach, they focused on the mechanism of wake flow. In others, active vibration control focusing on a vibration of the structure is studied [5,6]. Hirono et al.[7] applied the disturbance cancellation control to suppress the vibration of the cylinder. But there are few studies taking notice to model the lift force and construct a control system for vibration suppress.

In this study, we assume the lift force as a disturbance to apply an observer and combine two different control systems, feedback control and feedforward control [8]. In this control system, we obtain good control performance when the frequency of the disturbance model for the observer coincides with the vortex shedding frequency in simulation and experiment. When the model frequency differs from the vortex shedding frequency, performance of the control system falls down. To prevent this problem, we attempt to measure the frequency by FFT analysis and remodel the observer for the feedforward control. Simulations are carried out and the control performance is examined.

2. Modeling and Control System

Vortex-induced vibration of an elastically supported rigid circular cylinder in a cross flow is considered. The cylinder moves to the direction perpendicular to the cross flow and rotates around the center of gravity. The model for the analysis is illustrated in Figure 1. Fluid force and control force u exerts on the center of gravity of it and the bottom end of the cylinder in y-direction, respectively. Expressing the displacement and the torsion angle of the beam as \( y(x,t) \) and \( \phi(x,t) \), respectively, the equations of motion of the cylinder are given as

\[
M \left\{ \ddot{y}_1(l,t) + \frac{L}{2} \ddot{\phi}(l,t) \right\} - 2E_1I_1 \left[ 1 + c_1 \frac{\partial}{\partial t} \right] \gamma_1(l,t) = 2E_2I_2 \left[ 1 + c_2 \frac{\partial}{\partial t} \right] \gamma_2(l,t) - u + w
\]

\[
J \ddot{\phi}(l,t) + 2G_1I_{pl} \left[ 1 + c_3 \frac{\partial}{\partial t} \right] \phi_1(l,t) + 2G_2I_{pl} \left[ 1 + c_4 \frac{\partial}{\partial t} \right] \phi_2(l,t)
+ E_1I_1 \left[ 1 + c_5 \frac{\partial}{\partial t} \right] \gamma_1(l,t) L + E_2I_2 \left[ 1 + c_6 \frac{\partial}{\partial t} \right] \gamma_2(l,t) L = u \frac{L}{2}
\]

where \( E_i, I_i, G_i, I_{pl} \) represent young’s modulus, moment of inertia of area, modulus of transverse elasticity, polar moment of inertia of area, respectively, and \( c_{1-6} \) represent dumping coefficients. \( M \) and \( J \) are mass and moment of inertia of cylinder, respectively.
Assume the shear force and the inertia force of the translation motion as

\[ E_I \left(1+c_1 \frac{\partial}{\partial t}\right)y_1''(l,t) \equiv E_I \left(1+c_2 \frac{\partial}{\partial t}\right)y_2''(l,t), \tag{3} \]

\[ M \left(\ddot{y}_1(l,t) + \frac{L}{2} \ddot{\phi}(l,t)\right) \equiv M \ddot{y}_2(l,t), \tag{4} \]

respectively. Applying the geometrical continuity condition

\[ y_1(l,t) - y_2(l,t) = L \phi(l,t) = L \phi_2(l,t), \tag{5} \]

the equations of motion can be represented by the deflection \( y_1(x,t) \) and the torsion angle \( \phi_1(x,t) \) of the beam at the bottom end of the cylinder. Using eigenfunctions \( Y_{il}(x) \), \( \Phi_{il}(x) \) and time functions \( q_{il}(t), r_{il}(t) \) of \( i \) th mode, the deflection and the torsion angle of the beam can be approximated as

\[ y_1(x,t) = \sum_{i=1}^{\infty} Y_{il}(x) q_{il}(t), \tag{5} \]

\[ \phi_1(x,t) = \sum_{i=1}^{\infty} \Phi_{il}(x) r_{il}(t), \tag{6} \]

respectively. Consider first two modes and represent state vector as

\[ x = \begin{bmatrix} q_{11} & q_{11} & r_{11} & \dot{r}_{11} \end{bmatrix}^T, \tag{7} \]

state equation becomes

\[ \dot{x} = Ax + bu + dw. \tag{8} \]

The disturbance exerting on the cylinder has two aspects. One is transient disturbance and the other one is endurance disturbance. We apply feedback control to former aspect and feedforward control to latter aspect. Total control force consists of feedback control force \( u_f \) and feedforward control force \( u_g \) as follows:

\[ u = u_f + u_g. \tag{9} \]

We apply the optimal regulator to the feedback system. Feedback control force can be found by minimizing the quadratic performance index

\[ J = \int_0^\infty \left( x^T Q x + R u_f^2 \right) dt, \tag{10} \]

where \( Q, R \) are weight matrix and coefficient. With the feedback gains \( f \) derived from Eq.(10), feedback control force is given by

\[ u_f = -fx. \tag{11} \]

For the feedforward control system, we construct a minimal order observer to estimate the disturbance and determine the feedforward control force to cancel the disturbance. Assuming the disturbance as sine wave with vortex shedding frequency \( \omega_n \), it satisfies

\[ \dot{w} = A_w \ddot{w} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & 0 \end{bmatrix} \ddot{w}, \tag{12} \]
Then introduce design parameter $L$ and new state variable $\mathbf{z}$, estimated disturbance $\hat{\mathbf{w}}$ can be written as

$$
\dot{\mathbf{z}} = (A_d - LD)\mathbf{z} + [A_d L - L(A - bf) - LDL]x - Lbu_f,
$$

$$
\hat{\mathbf{w}} = \mathbf{z} + Lx, \quad L = \begin{bmatrix} 0 & l_1 & 0 & 0 \\ 0 & 0 & l_2 & 0 \end{bmatrix}, \quad D = [d \quad 0].
$$

(13)

Stability of the observer depends on the pole of $A_d - LD$. The pole of the observer has to be arranged more stable than the pole of the regulator. Feedforward control force is determined to cancel the estimated disturbance and combined state equations are

$$
u_f = -[1 \quad 0]\dot{\mathbf{w}} = -[1 \quad 0](\mathbf{z} + Lx),
$$

(14)

$$
\dot{x} = (A - bf - BL)x - Bz + Dw
$$

$$
\mathbf{z} = [A_d L - L(A - bf) - L(B - D)L]x + [A_d - L(B - D)]\mathbf{z}.
$$

(15)

3. Simulation and Experiment

3.1 Without frequency measurement

Experimental setup is shown in Figure 2. Cylinder and supporting beams were made of polycarbonate and steel, respectively. The parameters used in this study are $\rho = 7389\text{kg/m}^3$, $A_{12} = 9.0 \times 10^{-9}\text{m}^2$, $l = 0.177\text{m}$, $E_{12} = 142\text{GPa}$, $I_{12} = 6.75 \times 10^{-14}\text{m}^4$, $G_{12} = 49\text{GPa}$, $I_{p12} = 6.77 \times 10^{-11}\text{m}^4$, $c_{12} = 0.0003$ and $c_{13} = 0.0015$. Natural frequencies of the elastically supported cylinder are $f_1 = 4.99\text{Hz}$ for the first mode (translation mode) and $f_2 = 32.6\text{Hz}$ for the second mode (rotation mode).

Electro magnets were used as control actuator. They were set 15mm apart from fixed end of the plates and space between the magnet and beam was 1mm. Attracting force of electro magnet depends on a distance from the beam. We approximated the attracting force from experimental data $F = 1.360V^2$. Because of the limitation of amplifier, the maximum voltage of the magnet was 12V.

Weight matrix and coefficient were set to $Q = diag\{10^{-2}, 10^{-2}, 10^{-4}, 10^{-4}\}$, $R = 10$. The design parameter for the observer became $\{l_1, l_2\} = (0.220, 0.511)$. The pole of regulator was $\mu_{l2} = -0.65 \pm 31.4i$. By the foregoing condition between poles of regulator and observer, the pole of the observer of translation component was set to $\lambda_{l2} = -2 \pm 31.4i$.

Simulation and experiment were carried out at a lock-in region of the first mode of the elastically supported cylinder. Discrete model was derived from Eq.(15) with sampling time $T = 10\text{ms}$. Figure 3 illustrates the performance of feedback control and combined control. The solid line and the dashed line show the displacement with combined control and with feedback control, respectively. Combined control indicates sufficient performance in terms of the displacement suppression and the stabilization time.

The effect of the frequency error is shown in Figure 4. Triangle and square marks denote results of feedforward control and combined control, respectively. Combined control indicates sufficient performance in terms of the displacement suppression and the stabilization time.

The frequency range $\pm 1\text{Hz}$, both control systems suppress the vibration sufficiently and the deterioration will not occur. This might be happened because the cylinder will be forced to vibrate with the estimated frequency and the disturbance frequency will transit to the estimated frequency. The deterioration becomes large when the frequency error has negative value. There are two factors affecting to the deterioration of the control performance. One is the phase difference between the disturbance model and the real disturbance (Figure 5). Another is that the observer estimates the amplitude of the disturbance much larger than the real disturbance (Figure 6). As seen from Figure 5,
if the observer frequency deviates from the disturbance frequency, the phase difference increases up to 90[deg].

When the error has positive value, the observer estimates the disturbance larger than that of estimated by the observer with negative error. So that the sufficient control force is applied to the cylinder to suppress the vibration of the cylinder when the error is positive.

![Graph](image1.png)

**Figure 3** Time responses of elastically supported cylinder: (a) Simulation result; (b) Experimental result

![Graph](image2.png)

**Figure 4** The effect of frequency error of disturbance model: (a) Simulation result; (b) Experimental result

![Graph](image3.png)

**Figure 5** Phase of estimated disturbance

![Graph](image4.png)

**Figure 6** Amplitude of estimated disturbance

### 3.2 With frequency measurement

When the frequency of the disturbance model in Eq.(10) is different from the frequency of the real disturbance, deterioration of the control performance occurs. To prevent this problem, we attempt to measure the frequency by FFT analysis and remodel the observer for the feedforward control. The number of data for FFT analysis is 1024.

Figure 7 and Figure 8 show the time responses of the displacement applying the feedforward control and the combined control, respectively. The disturbance frequency is  $f_d=5.00\text{Hz}$ . Each control is applied at $t=10\text{s}$ with the frequency of the observer $f_r=4.70\text{Hz}$.

For $t=10\text{~s}$ to $40\text{~s}$, the feedforward control could not suppress the displacement sufficiently because the observer could not estimate the disturbance adequately. Due to the feedback control, the displacement of the cylinder applying the combined control is much less than that applying the feedforward control. But still sufficient performance could not be obtained. As for the rotation angle, the amplitude became larger than that without control. The major cause of the augmentation of the rotation angle is that the control force was put on the bottom.
of the cylinder only though the disturbance was assumed to convert on the center of gravity of the cylinder. Comparing the control voltage of the feedforward control and the combined control, the control voltage of the feedforward control is larger than that of the combined control due to the feedback control force. The control voltage of the feedforward control was 3.32 V and that of the combined control was 2.84 V.

At $t = 40s$, FFT analysis was carried out. Nyquist frequency was 50 Hz as the simulation was carried out with sampling period $T = 10\text{ms}$. This is enough to measure the frequency because the frequency of the second mode of the system is 32.6 Hz. Figure 9 and Figure 10 illustrate the power spectrum density of the displacement. The measured frequency from the displacement of each control was $f_s = 4.99\text{Hz}$. After remodeling the observer with the measured frequency, control performance advanced much better. There is a slight difference between the measured frequency and the disturbance frequency so that the feedforward control could not suppress the vibration of the cylinder completely. On the contrary, the combined control suppressed the vibration of the cylinder so well. Rotation angles are almost same in both control systems. After remodeling the observer, the rotation angle enhanced along with the augmentation of the control voltage. For combined control, rotation angle could not be

![Figure 7 Simulation results (Feedforward control only)](image7)

![Figure 8 Simulation results (Combined control)](image8)

![Figure 9 Power spectrum density (Feedforward control only)](image9)

![Figure 10 Power spectrum density (Combined control)](image10)
suppressed enough though the feedback control was applied. The lock-in region of the first mode (translation mode) of the cylinder was focused here and the weight matrix of the feedback control was selected to suppress the displacement selectively. The control voltages after applying the FFT analysis have approximately the same magnitude. This is due to the contribution of the feedforward control. The feedforward control suppresses the vibration of the cylinder drastically even if the slight difference between the disturbance frequency and the measured frequency existed, so that the feedback control force is diminished.

4. Conclusions

Disturbance cancellation control that adapts to the frequency variation by remodeling the observer with measured frequency has been investigated. When the frequency of the disturbance model differs from that of the real disturbance, the observer cannot estimate the disturbance correctly and causes a deterioration of the control performance. To prevent this problem, the observer is reconstructed with measured frequency. FFT analysis is used to measure the disturbance frequency. Remodeling the observer with the measured frequency can get the sufficient performance out of the feedforward control. The control performance depends on the accuracy of the frequency measurement. Even if the measured frequency has a slight difference with the disturbance frequency, the combined control manifests sufficient performance.

5. References